Tutorial Information for MATH 2020A (2024 Fall)
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1. Let $f(x,y) = \dfrac{\ln x}{xy}$ and the rectangle region $R = \{(x,y) : 1 \le x \le e, 1 \le y \le 4\}$. Evaluate the integral

$$\iint_R f(x,y)\,\mathrm{d}A.$$

Solution: $\ln 2$

2. Let $\Omega \subset \mathbb{R}^3$ be the solid bounded above by the paraboloid $\{(x,y,z) : z = x^2 + y^2\}$ and below by the square region $R = \{(x,y,0) : -1 \le x \le 1, -1 \le y \le 1\}$. Sketch this solid and find its volume.

Solution: $\frac{8}{3}$

3. (a) Let $y > 0$ be a fixed number, evaluate the integral $I_1(y) = \int_0^2 \frac{y-x}{(x+y)^3}\,\mathrm{d}x$.

(b) Can $I_1(y)$ as a function of y be extended to be a continuous function defined on $[0,\infty)$?

(c) If (b) is true, then $I_1(y)$ is integrable on $[0,\infty)$. Calculate the integral $\int_0^1 I_1(y)\,\mathrm{d}y$.

(a') Let $x > 0$ be a fixed number, evaluate the integral $I_2(x) = \int_0^1 \frac{y-x}{(x+y)^3}\,\mathrm{d}y$.

(b') Can $I_2(x)$ as a function of x be extended to be a continuous function defined on $[0,\infty)$?

(c') If (b') is true, then $I_2(x)$ is integrable on $[0,\infty)$. Calculate the integral $\int_0^2 I_2(x)\,\mathrm{d}x$.

(d) Let $f(x,y) = \frac{y-x}{(x+y)^3}$, conclude the following two iterated integrals are not equal:

$$\int_0^1 \int_0^2 f(x,y)\,\mathrm{d}x\,\mathrm{d}y \quad \text{and} \quad \int_0^2 \int_0^1 f(x,y)\,\mathrm{d}y\,\mathrm{d}x.$$

Does this contradict Fubini's theorem? Why?

Solution: (a)$\frac{2}{(y+2)^2}$; (b)Yes; (c)$\frac{1}{3}$;
(a')$\frac{-1}{(x+1)^2}$; (b')Yes; (c')$-\frac{2}{3}$;
(d)Not a contradiction, becasue $f(x,y)$ is not continuous on the closed region.

4. Let $R \subset \mathbb{R}^2$ be the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$, and $x = 2$.

   (a) Sketch the region R.

   (b) Let $f(x,y) = \frac{x}{y}$, evaluate the integral $\iint_R f(x,y)\,\mathrm{d}A$.

Solution: $\frac{3\ln 2}{2}$

5. Evaluate the integral

$$\int_0^\pi \int_x^\pi \frac{\sin y}{y}\,\mathrm{d}y\,\mathrm{d}x.$$

Solution: 2